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Modelling Real Options: the case of a small hydro investment project

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1. THE REAL OPTIONS APPROACH

1.1 – Real Options Approach

The Real Options Theory is perceived as the only method of assets valuation that recognizes the interaction between the three factors that characterize the nature of investments: irreversibility, uncertainty and flexibility in timing (Dixit and Pindyck, 1994).

In a context of uncertainty and flexibility, the evaluation of an investment must take into account the possibility of response to future operating conditions. The technical evaluation of real options has the capability to account for this investment flexibility (Soares et al., 2008). The following figure represents a matrix that relates the uncertainty and flexibility with the methodologies that evaluate risk and uncertainty in project analysis.

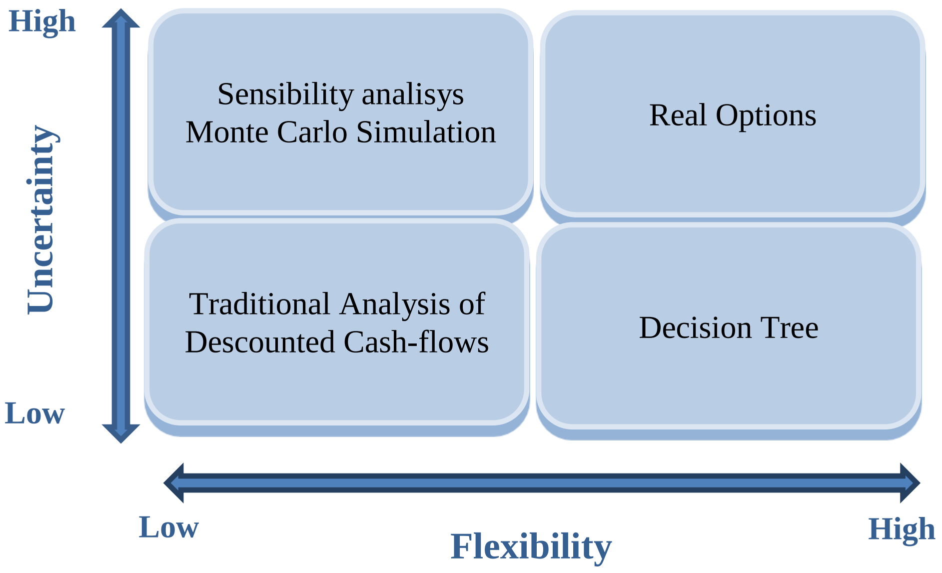

Figure 1 - Uncertainty and Flexibility

Source: Elaborated by the author based on Soares et al., 2008

As it has been claimed in this paper, this matrix shows that the ROA is the best method of investment evaluation when flexibility is incorporated into the investment and there is a high level of uncertainty.

The ROA results from developments in studies of Financial Options. Thus, for explaining the application of Real Options, it is necessary to establish theoretical concepts that also resulted in Financial Options.

1.1.1. *Financial Options and Real Options*

A financial option is an asset that gives the holder the right but not the obligation, to buy (call option) or sell (put option) a certain amount of a particular asset (underlying asset), to a pre-determined fixed price (exercise price), within a certain period or established date (Soares et al., 2008).

In 1973, Miller-Fisher Black and Myron Scholes derived the first mathematical formula for pricing options of purchase shares (call options) of the european type (Black and Scholes, 1973). In their article, Black¬Scholes start from a non-arbitrage premise (proposed by Modigliani and Miller) to develop an equilibrium model that involves a risk-free portfolio, whose return could be represented by risk-free rate.

The Black-Scholes model (1973) takes into account the following assumptions:

1. The risk-free rate is known and constant over time;
2. The asset pays no dividends;
3. The option can only be exercised at the time of maturity (Option of European type);
4. There are no transaction costs when buying or selling an asset or derivate;
5. It is possible to invest any fraction of assets or derivates to the risk-free interest rate;
6. There are no penalties when making short-selling;
7. The model derives from the concept that asset price of an option has a continuous stochastic behaviour in the form of Geometric Brownian Motion (GBM) according to the following equation:

$$\frac{dS}{S} = \mu dt + \sigma dz \qquad \textbf{(1)}$$

Where:

dS: Variation of S (underlying asset price) at time dt;

μ: A mathematical expectation of the instantaneous return rate of the underlying asset;

σ: The instantaneous standard deviation of return on the underlying asset;

dz: A standard process of Gauss-Wiener[1].

The Black-Scholes equation for European call option is:

$$\boldsymbol{c = SN(d_1) - Ke^{-r\tau}N(d_2)} \tag{2}$$

Where:

$$\boldsymbol{d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}} \tag{3}$$

and,

$$\boldsymbol{d_2 = d_1 - \sigma * \sqrt{\tau}} \tag{4}$$

Where:

$N(d)$: Function of Cumulative normal distribution;

ln: Natural logarithm;

S: Stock price;

K: Exercise price;

r: Risk-free rate with continuous capitalization;

τ: Time to expiration;

σ: The volatility of underlying asset.

The Black-Scholes equation for European put options is easily deduced from the previous equation through "put-call parity[2]". Considering that p is the value of the put option of an asset in time *t*, we have:

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[1] **Wiener Process:** A stochastic process $W_t = \{W(t), t \geq 0\}$ defined in a probability space (Ω, F, P) is a Wiener process if:
1. for $s \geq 0$ and $t > 0$, the random variable $W_{t+s} - W_s$ has a normal distribution N(0,t);
2. for $n \geq 1$ and $0 \leq t_0 \leq \cdots \leq t_n$, the random variable $W_{tr} - W_{tr-1}$ is independent;
3. $W_0 = 0$;
4. W_t is continuous for $t \geq 0$.

$$\boldsymbol{p = Ke^{-r\tau}N(-d_2) - SN(d_1)} \qquad \textbf{(5)}$$

While European options can only be exercised at maturity date, the American options can be exercised at any time until the maturity date of an option. These American and composed options require for their valuation, the use of numerical methods, such as binomial tree developed by Cox, Ross and Rubinstein (1979). According to these authors, this development comes from a simple and efficient procedure for options evaluation, allowing by essence of its construction, the optimal premature exercise of an option. For these options, we must decide at every instant, which of two actions is most beneficial: exercise option in advance or wait for maturity date.

In this model it is assumed that the period to an option maturity can be divided in discrete periods, whose dimension will be represented by Δt, assuming in each period a given behaviour for the underlying asset price. Each time interval Δt, the underlying asset price is multiplied by an random coefficient μ or d. This random coefficient is the rate of variation price in the underlying asset, which can be ascending (μ) or descending (d), reflecting the favourable and unfavourable conditions in the market. These multiplicative factors depend on volatility (σ) and size of time interval (ΔT). The **Figure 1** is a binomial tree of evolution for the underlying asset price, where the nodes on the right represent the distribution of possible future values for the underlying asset in option maturity.

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[2] The ***put-call parity*** is resulted by (Soares et al., 2008):

- a composed portfolio by a *long position* in an unit of underlying asset;
- a *short position* in a call option (meaning it had sold the asset without owning, that is sold to uncovered);
- and a *long position* in a put option,

In maturity date of the options is always has the value of exercise price. Therefore, in the absence of arbitrage opportunities, the portfolio value at any point in time is the value of the exercise price discounted by risk-free interest rate.

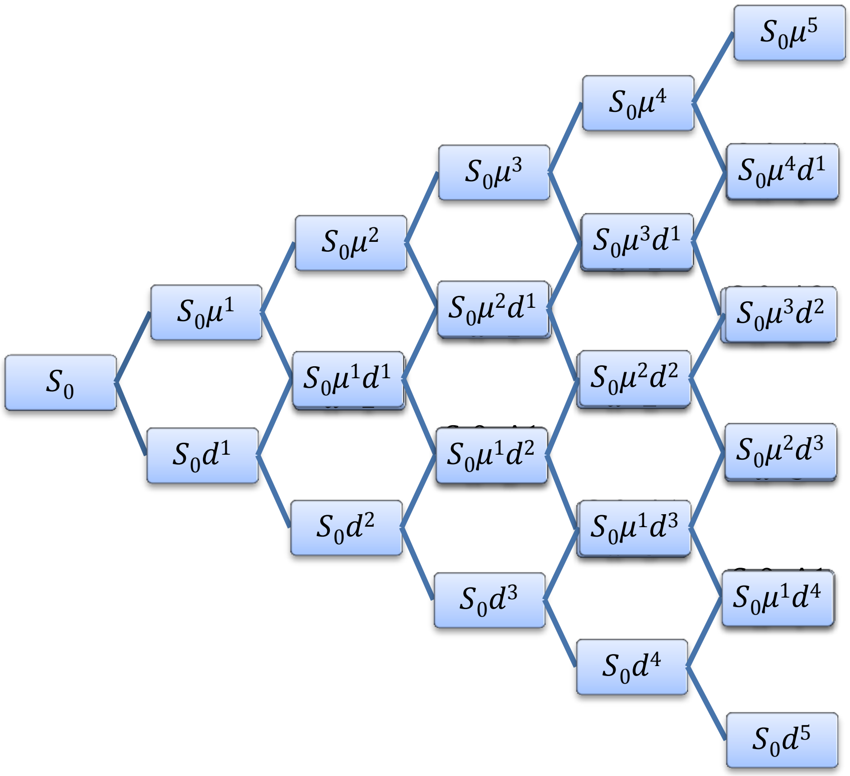


Figure 1 - Binomial tree of evolution for the underlying asset price
Source: Elaborated by the author based on Soares et al., 2008

The ascent and descent coefficient values of the stock in each time interval, d, respectively, are given by:

$$\boldsymbol{\mu = e^{\sigma\sqrt{\Delta t}}}\ \textbf{(1)}\ \text{ and }\ \boldsymbol{d = e^{-\sigma\sqrt{\Delta t}}}\ \textbf{(6)}$$

The probability of stock price increase or decrease is given by risk-neutral measure by p and $q = 1 - p$, respectively. This probability is given by the following equation:

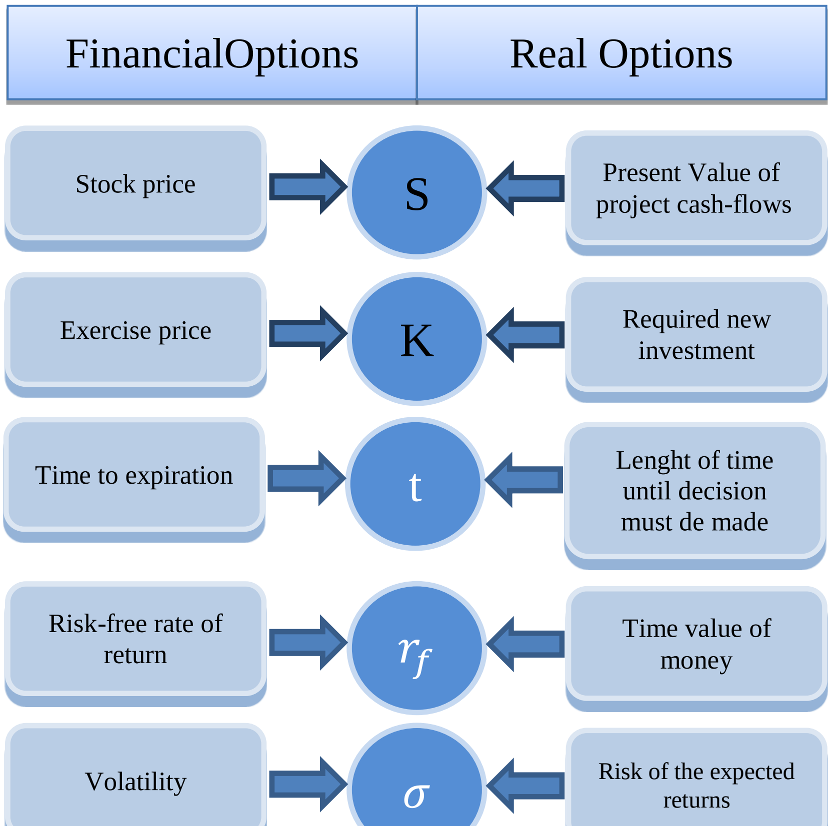
$$\boldsymbol{p = \frac{e^{r_f \Delta T} - d}{u - d}} \qquad \textbf{(7)}$$

When these parameters are determined, it is possible to get values for each option through an option evaluation tree. In this tree is represented each obtained gain for stock price. In the case of a call option, this value is given by the maximum difference between value of the underlying asset and exercise price and zero, i.e., *max (S-K,0)*, while in the case of a put option, the value corresponds to the maximum difference between exercise price and stock price and zero, i.e., *max (K-S,0)*. From the option value in the right nodes of the tree, it is calculated the other values applying the neutral probability on each pair of values vertically adjacent, represented mathematically by the following equation:

$$\boldsymbol{c_t = \frac{p{c_\mu}^{t+1} + (1-p){c_d}^{t+1}}{e^{r\tau}}} \qquad \textbf{(8)}$$

From the current stock price we determine the different trajectories that it can follow in time until it reaches maturity. For the option value it is adopted an opposite route, from right to left, based on the prices defined in each node.

Identically to financial options, the real options are the right but not the obligation to take an action that affects a real physical asset, at a pre-determined cost, during a pre-established time (Soares et. al., 2008). Therefore, the following figure represents the real and financial options determinants:



Source: Elaborated by the author based on Sanisio, 2002

1.2. Real Options typologies

When dealing with an investment project several options can be exercised, like the option to defer investment, cancel new steps of investment, change the scale of production (expand, contract, temporarily shut down, restart), abandon by residual value of the project, change uses (inputs and outputs) and growth options (Trigeorgis, 1995). These typologies of real options can be classified by flexibility offered in accordance with the following taxonomy:

Table 1 - Types of Real options

Delay Option	It is an American call option found in most projects where exists the possibility to postpone the beginning of investment
Abandon Option	The abandon option of a project for a fixed price (even if that price declines over time) is formally an American put option
Contraction Option	The contraction option (reduce size) of a project, by selling a fraction of this project for a fixed price, is also an American put option
Option for growth and expansion	The expand option of a project, paying more to increase it, is an American call option
Compound Options	There are also options on options, called composited options. The investments planned in phases fall into this category. In these cases it is possible to stop or delay the project in the end

	of each phase. Thus, each phase is a contingent option to previous exercise of other options: an option on options.

Source: Elaborated by the author

1.2.1. Delay Option

The option to delay a project provides a right, but not an obligation to its holder, to make the investment in the next period, and only performed if the value of investment of the next period exceeds the necessary investment on the current date.

In other words, this option corresponds to an American option, allowing the postponement of an investment decision during a given time. Since, the investment decision implies not exercising the option of waiting, this value of option loss is similar to an additional opportunity cost, which justifies investment only when the NPV exceeds the value of the deferral option (Trigeorgis, 1995).

The delay option confronts the gains of uncertainty resolution and obtaining of additional information, with the costs from project deferral. These costs are reflected, on the one hand, in competitive position, since the deferral may cause partial or total loss of investment value due to the actions of competitors; and on the other hand, loss of positive cash-flows generated by an investment that was not undertaken (Soares et al., 2008).

1.2.2. Abandon Option

In an unfavourable situation for project viability, the abandon option can be exercised in order to give additional value to the investment when there is liquidation of its assets (Soares et al., 2008).

The first option type happens when an investment is divided in such a way that it can be abandoned at any time, since the costs are not concentrated in one period. In this case, it is a situation of sequential investment, in which are determined a series of options on options called Compound Options.

The second type of option consists in the complete abandonment of the project, only getting the amounts for capital expenditures that have not been realized or its residual value.

According to Brealey and Myers (2003), the abandonment of project provides a partial insurance against investment failure. This option is equivalent to an American put option, in which the exercise price corresponds to the liquidation value of investment assets.

1.2.3. Contraction Option

If the conditions are unfavourable in a given market conditions, it is possible to reduce the production scale, reserving part of the planned investment expenditures. This capability is similar to a put option on part of the project, with an exercise price equal to the potential costs saved.

1.2.4. Option for growth and expansion

Contrary to the previous points, this option is exercised in cases of favourable market conditions for the project. This option is identical to an American call option to acquire an additional part of the project, requiring an accompaniment cost (exercise price) ((Trigeorgis (1995).

This option allows promoting pilot-projects for new technologies, which even with negative NPV, should be performed, because these projects can put on the market new successful products or processes. In other words, in these cases, the projects that were initially rejected by traditional assessment methods should be implemented (Soares et al., 2008).

2. CASE STUDY: APPLICATION OF REAL OPTIONS TO A SMALL HYDRO INVESTMENT PROJECT

2.1. Small hydro investments

The term mini-hydro plant differs from large hydro plant, since the first, due to its small environmental impact, is considered a renewable technology. As for the second, although, it uses a renewable resource, it produces non-negligible effects on the environment, which make their classification as a renewable resource technology problematic.

Mini-hydro plants use the following classification recommended by UNIPEDE relatively to installed capacity and height of fall:

Table 2- Classification of hydro plant by installed Capacity

Designation	***P(MW)***
Small-hydropower plant	<10
Mini-hydropower plant	<2
Micro-hydropower plant	<0,5

Source: Elaborated by the author based on UNIPEDE, 2009

Table 3 - Classification of hydro plant by height fall

Designation	***H(m)***
Low fall	2-20
Average fall	20-150
High fall	>150

Source: Elaborated by the author based on UNIPEDE, 2009

The mini-hydro plants are very criticized for their impact on the ecosystem. First, they avoid the connection between upstream and downstream of the installation, having negative consequences, such as the block of passages and protection for fishes, interruption of sediment transport and impact on the landscape in areas little explored.

Systems of mini-hydro plants convert the potential and kinetic energy of water in electricity movement, using a turbine that drives a generator. As the water runs from a high point to a lower zone, as in rivers and waterfalls, the energy is transported that can be exploited by the system of mini-hydro plant.

A constant flow of water is critical to the success of a project for a mini-hydro. The energy available from a turbine is proportional to the amount of water that passes through the turbine per unit of time (i.e., flow), and the vertical difference between the turbine and the water surface to water inlet. Like most of the cost of a project for a mini-hydro results from construction expenses and purchase of equipment, this investment can generate large amounts of electricity with very low operational costs and modest maintenance costs for 50 years or more (RETScreen International, 2005).

Comparatively with other technologies from renewable sources, these plants have a high technological efficiency, due to their maturity level, which reduces significantly the technological risk. Relatively to intermittency of generation, this technology has variation rates and low intermittency, with small variations from day to day. Moreover, as mentioned earlier, their resource (water) is easily predictable, which reduces the uncertain amount of energy generated.

The following figure shows the main components of a mini-hydro plant:

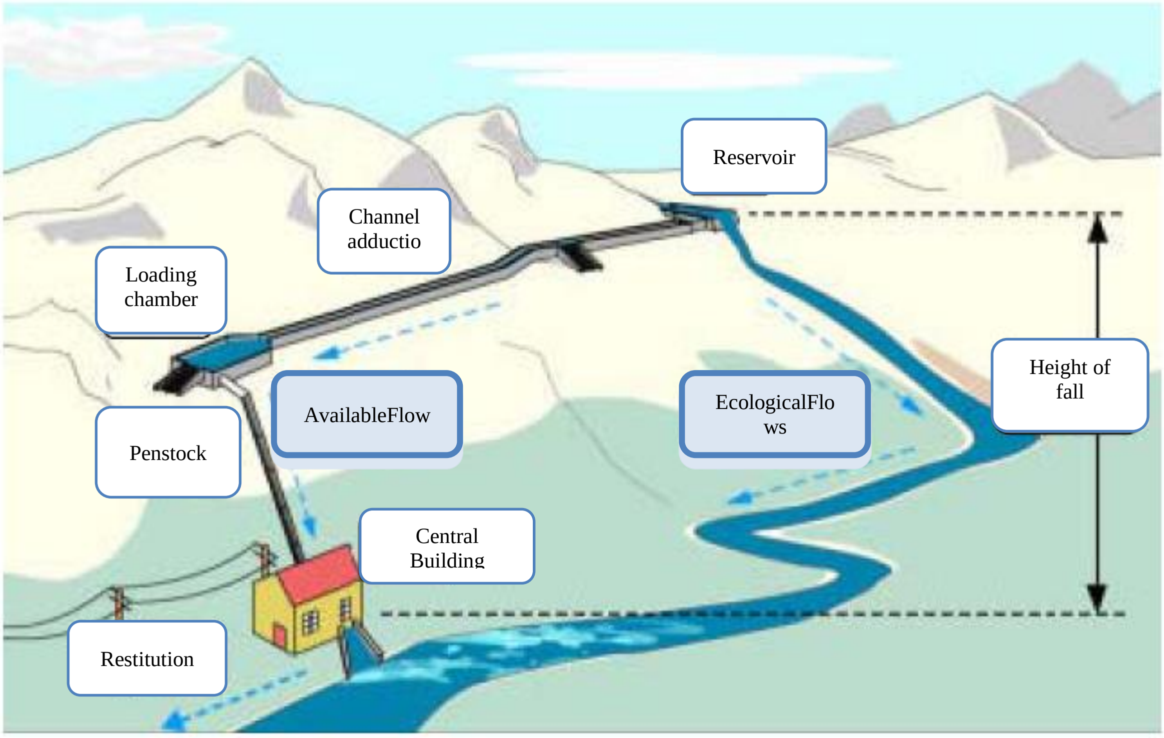


Figure 3 – Components of Mini-hydro plant
Source: Camus and Eusébio (2006)

According to the report of analysis of clean energy projects RETScreen International (2005), some authors, usually consider four stages of engineering work required to develop a project for a hydroelectric plant. These steps of project are represented in the following figure:

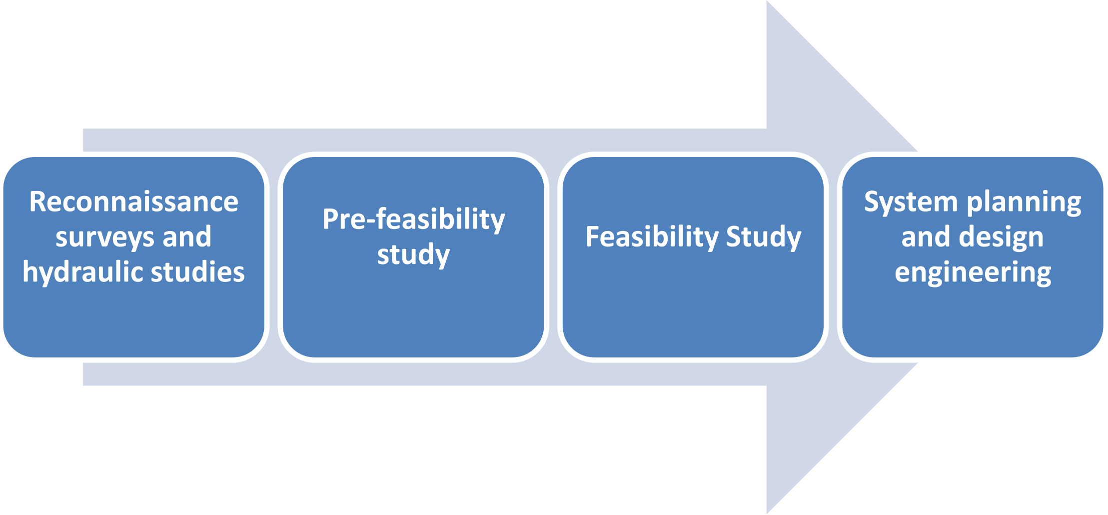


Figure 5 - Main stages of hydropower projects

Source: elaborated by the author

Reconnaissance surveys and hydraulic studies: This first phase of work usually covers numerous sites and includes: map study, delineation of drainage basins; preliminary estimates of flow and flooding; and one day visit to each location (by an engineer and project geologist or geotechnical engineer); preliminary layout; cost estimates (based on formulas or computer data); a final classification of sites based on the energy potential, and a cost index.

Pre-feasibility study: Work on chosen site or sites include: mapping the location and geological investigations; recognition for a suitable borrow areas (eg, sand and gravel); a preliminary layout based on known materials to be available; primary selection of characteristics of main project (installed capacity, type of development, etc.); a cost estimate based on major amounts; identification of possible environmental impacts; and elaboration of a single report on each site.

Feasibility Study: Work continues on the selected site with a major program of foundation investigation; design and testing of all borrow areas; estimate of deviation, design, and probable maximum flood; determination of energy potential for a range of heights dams and installed, determining the project design earthquake and maximum credible earthquake; design of all structures in sufficient detail to obtain quantities of all items that contribute more than about 10% to the cost of individual structures; determination of the dewatering sequence and project plan; optimizing the layout of the project, water levels and components; production of a detailed cost estimate; and finally, an economic and financial evaluation of the project, including an assessment of the impact on the existing electrical wiring, along with a feasibility report.

System planning and design engineering: This work should include studies and final design of the transmission system; transmission system integration; integration of the project to the power grid to determine the precise mode of operation; production of tender drawings and specifications; reviewing proposals and detailed design of the project; production of detailed construction drawings and review of drawings of the equipment manufacturer.

However, for a mini-hydro, the engineering work is often reduced to three stages, with a lower level of detail in order to reduce costs. Generally, a preliminary investigation is conducted, which combines the work involved in the first two phases described above. While reducing the engineering work, increases the risk of the project not being financially viable, which can usually be justified, due to the reduction of costs associated with smaller projects (RETScreen International, 2005).

2.2. Case study: a brief description

In this point it will be analysed a former investment valuation on a hydroelectric plant where were used traditional methodologies of project valuation and then, proceed to a practical application of the ROA to this case study. Thus, the phases of the mentioned project will not be assessed in this study, since they have been already finished. Only the phases related directly to the economic viability of the project, such as the economic and financial data and production estimates, will be in the scope of this study.

The case study represents an investment project of a mini-hydro plant with an installed capacity of 500 kW, resultant to a capture usage of low dropout (10.5 m), with a plant built on river margins, besides the concrete reservoir. The lifetime of the project is 50 years, which corresponds to the lifetime of the turbine and generator. The lifetime of the transformer is 25 years.

This project presents the following characteristics:

Table 4- Characteristics of the mini-hydro plant

Turbine Type	Kaplan with vertical axis
Nº of turbines	1
Generators	Asynchronous three-phase 400V
Nº of generators	1
Income generator	95%

Transformers	400V/15kV
Nº of transformers	1
Income of transformers	90%
Capacity of each turbine (kW)	500
Capacity of project (kW)	500
Interconnection line (km)	line de 15Kv with 10 km
Average annual generation (kWh)	1.332.808

Source: elaborated by the author

The project begun in 2006, and the start-up was the end of that year.

At the time of the economic assessment, the project costs were assumed to be following:

Table 5 - Investment Cost (%)

Investment Costs	***Percentage of Total***
Transformers	14,46%
Generators	10,24%
Turbines	10,24%
Electromechanical equipment	14,46%
Construction	24,10%
Line of 15kv	12,05%
Study and Project	2,41%
Cost of land and expropriation	12,05%

Source: elaborated by the author

Table 6 - Operating and Maintenance Costs (%)

Operating & Maintenance Costs (Annual)	***Percentage of total***
Years 1 a 50	2,48%
Maintenance year 10	4,97%
Maintenance year 20	4,97%
Maintenance year 30	4,97%
Maintenance year 40	4,97%
New transformers after 25 years	77,64%

Source: elaborated by the author

Regarding the financing of the project, there is an incentive program that funds 40% of the investment, being financed up to 1000 €/ kW. The equity of the company support is 25% of the investment and the remaining 35% are obtained by use of bank credit. The first 300 €/kW of incentive are not refundable, and the remainder must be repaid, without interests, in nine years with a waiting period of 3 years (i.e. from the 4th to the 9th year in annual constant payments). The bank financing is a 10 year credit, repayable through constant annual payments, with a 6.5% interest rate, from the date of entry to the operation of the plant. The opportunity cost of capital is considered 10%.

2.3.The economic evaluation of the project under a traditional approach: critical analysis

In this subchapter, it will be undertaken a critical analysis of the assessment made, focusing the following points:

- Calculation of energy produced;
- Value of the energy sales/year;
- Inflation rate;
- Depreciations;
- Rate of capital cost;
- NPV, IRR e Payback.

This project has been assessed from three main traditional methods: NPV, IRR and Payback. The main results obtained with this analysis were the following:

Table 7 - Results of project

Energy Produced (kWh/year)	1.332.808
Remuneration of energy (€)	9.6672
NPV (€)	51.371
IRR (%)	11,22%

Payback (years)	35

Source: Elaborated by the author

- **Energy produced**

The hydrological study was conducted for flow distribution based on the values of the monthly average flow, measured in a hydrological station located 1000m upstream of where is installed the mini-hydro, with a catchment area of 200 km2.

The monthly average flows presented are:

Table 8- Monthly average flow (m^3/s)

	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SET
"1966/67"	7,8	9,2	3,2	6,5	23	9,1	2,9	9,5	1,8	0,38	0,15	0,2
"1967/68"	0,4	2,9	1,1	0,87	21,75	5,7	8,9	1	1,4	0,24	0,06	0,72
"1968/69"	1,06	14	20	32	29	57	6,4	11	3,9	1,1	0,22	1,1
"1969/70"	0,83	2,2	1,6	8,5	7,2	2,7	1,6	4	1,3	0,19	0,05	0,02
"1970/71"	0,1	1,1	1,28	12,32	5,25	3,85	14,67	10,67	7,13	5,54	2,09	0,69
"1971/72"	1,31	0,73	0,91	4,74	37,44	11,66	3,99	1,84	0,76	0,15	0,04	0,17
"1972/73"	2,83	6,18	14,58	22,21	5,3	2,06	0,57	9,85	2,61	0,71	0,28	0,24
"1973/74"	1,72	1,75	3,02	20,98	20,08	4,6	2,4	2,29	10,36	2,46	0,37	0,34
"1974/75"	0,37	2,26	1,55	4,26	7,71	13,81	2,92	1,59	0,89	0,22	0,04	0,45
"1975/76"	1,3	1,24	1,98	1,19	4,86	2,44	2,06	1,63	0,38	0,58	0,19	0,53
"1976/77"	6,37	6,52	8,79	22,38	45,19	10,38	5,46	1,97	1,71	0,37	0,15	0,11
"1977/78"	2,31	1,59	39,3	12,03	61,98	23,97	4,68	6,96	1,84	0,39	0,07	0,04
"1978/79"	0,44	1,32	39,3	24,05	72,96	23,45	15,64	3,51	1,6	0,53	0,09	0,05
"1979/80"	5,16	3,16	4,62	5,52	10,18	7,82	5,54	5,59	1,83	0,31	0,11	0,07
"1980/81"	0,35	1,69	1,06	0,82	1,23	3,75	5,76	5,72	1,62	0,14	0,03	0,15
"1981/82"	3,13	0,58	23,76	14,92	7,26	3,2	1,59	0,98	0,8	0,11	0,04	0,65
"1982/83"	1,78	6,74	9,43	3,09	10,17	4,47	14,39	22,21	3,37	0,84	0,71	0,27
"1983/84"	0,28	6,09	19,7	6,57	5,28	7,08	8,01	3,42	2,23	0,43	0,06	0,13
"1984/85"	5,2	28,05	13,06	23,58	45,81	9,7	9,76	3,25	4,21	0,67	0,21	0,07

"1985/86"	0,16	1,37	14,49	10,25	21,28	8,79	3,31	1,9	0,41	0,06	0	3,43
"1986/87"	0,68	3,5	3,24	7,25	15,84	6,32	7,79	2,15	0,75	0,59	0,06	2,71
"1987/88"	6,77	3,99	13,81	31,47	27,02	3,69	4,44	10,08	4,18	2,69	0,34	0,14
"1988/89"	1,08	1,71	1,44	1,15	2,71	4,2	5,07	2,32	2,99	0,2	0,11	0,05
"1989/90"	0,14	13,36	72,21	14,1	15,11	3,33	4,42	1,65	0,64	0,17	0,1	0,12

Hydrological studies provide the probability of flows (usually daily mean values) during the year. It is necessary an analysis of records over several years in order to calculate the water resources during the life of the mini-hydro (Camus and Eusebio, 2006).

It can be supplied data relative to daily and monthly average flows to calculate: the average energy produced; flows in dry, wet and normal years to study scenarios; flood flows, for the design of water retaining structures and spillways; ecological flows, to calculate the available flow (Camus and Eusebio, 2006).

The primary objective of the hydrologic analysis designed to support feasibility studies of hydroelectric power plants is, therefore, to obtain the call duration of the flow-duration curve. This curve is a mean curve supported by observations made over several years and its significance will be greater, the longer the time period required for its construction (Castro, 2002).

This way, the values considered for the period 1966 to 1990, used to calculate a forecast of production for a project that started in 2006, may not be enough. Given the climatic changes over the past 40 years, it is important that these values were updated, at least until the beginning of the twenty-first century. Moreover, it is recommended that these data matches 30 to 40 years (and Camus and Eusebio, 2006; Castro, 2002).

➢ Value of energy sold annually

The tariff calculation is performed based on assumptions and does not take into account the true values of each part of the tariff. The remuneration value of generated energy is calculated using the following formula:

$$VRD_m = [KMHO_m * [PF(VRD)_m + PV(VRD)_m] + PA(VRD)_m * Z] * \frac{IPC_{m-1}}{IPC_{ref}} * \frac{1}{(1-LEV)} \quad \textbf{(9)}$$

Where:

$\mathrm{VRD_m}$:Monthly remuneration applicable to central of Renewable Producers;
$\mathrm{KMHO_m}$: It is a coefficient that modulates the values of$\mathrm{PF(VRD)_m}$, $\mathrm{PV(VRD)_m}$ and $\mathrm{PA(VRD)_m}$as a function of time in which electricity has been provided;
$\mathrm{PF(VRD)_m}$: Fixed portion of remuneration (capacity) applicable in the month m;
$\mathrm{PV(VRD)_m}$: Variable portion of remuneration (energy) applicable in month m;
$\mathrm{PA(VRD)_m}$: Environmental portion of remuneration in month m;
Z: Additional coefficient that reflects the characteristics of the resource and technology used;
$\mathrm{IPC_{m-1}}$: Consumer price index, excluding housing, on the Continent, in the month m;
$\mathrm{IPC_{ref}}$: Consumer price index, excluding housing, on the Continent, in the month prior to the start of power supply;
LEV: Losses in transmission and distribution avoided by the renewable central.

When reviewing the calculation of the first part of the equation (KMHO), the value is considered equal to 1. However, although in the licensing process, the renewables have the possibility to decide if they prefer or not the tariff modulation translated by the coefficient KMHO, the hydro plants have obligatory modulation. Thus, the value should not be considered 1, but should be calculated based on the assumption of legislation and the following formula:

$$KMHO = \frac{KMHO_{pc} * ECR_{pc,m} + KMHO_{v} * ECR_{v,m}}{ECR_{m}} \tag{10}$$

Where:
$\boldsymbol{KMHO_{pc}}$**:** Factor that represents the modulation corresponding to full and peak hours, which have the value of 1.15 for the hydro plants;
$\boldsymbol{ECR_{pc,m}}$ $(\boldsymbol{kW/h})$**:** Renewable electricity produced by the plant in full and peak hours and end of month *m*;
$\boldsymbol{KMHO_{v}}$**:** Factor that represents the modulation corresponding dumped hours, which have the value of 0.80 for the hydro plants;
$\boldsymbol{ECR_{v,m}}(\boldsymbol{kW/h})$***:*** Renewable electricity produced by the central in dumped hours of the month *m;*

$\boldsymbol{ECR_m(kW/h)}$**:** Renewable electricity produced by the plant in the month *m*.

The fixed part $\boldsymbol{PF(VRD)_m}$ is associated to the remuneration related to capacity guarantee provided by the renewable plant, and it is calculated by following equation:

$$\boldsymbol{PF(VRD) = PF(U)_{ref} * COEF_{pot,m} * POT_{med,m}} \tag{11}$$

Where:

$\boldsymbol{PF(U)_{ref}}$ is the unit value of reference for $PF(VRD)$, which:

- Must correspond to the monthly investment unit cost in new production facilities, which construction is avoided by a renewable energy plant, that ensures the same level of capacity that would be provided by a new production facility;
- It's value is 5,44 € (kW/h);
- It will be used, in each plant, during all periods in which the remuneration set by VRD is applied.

$\boldsymbol{COEF_{pot,m}}$: Dimensionless coefficient that reflects the plant contribution of renewable in the month *m* to guarantee capacity provided by the public network.

$\boldsymbol{POT_{med,m}}$: Average capacity available (declared) by renewable plant to the public network in month *m*, expressed in kilowatts.

The variable part of remuneration $\mathrm{PV(VRD)_m}$ is linked to the energy delivered by PRE-R, and is calculated as follows:

$$\boldsymbol{PV(VRD)_m = PV(U)_{ref} * ECR_m} \tag{12}$$

Where:

$\boldsymbol{PV(U)_{ref}}$ is the unit value of reference for $PV(VRD)$, which:

- Must correspond to the operation and maintenance costs that would be needed to exploit the new production facilities, which construction is avoided by the renewable plant;
- It's value is €0,036 kW/h;
- It will be used, in each plant, during all periods in which the remuneration set by VRD is applied.

The environmental part $\mathrm{PA(VRD)_m}$ values the environmental benefits provided by the renewable plant, and it is calculate by the following formula:

$$\boldsymbol{PA(VRD)_m = ECE(U)_{ref} * CCR_{ref} * ECR_m} \qquad \textbf{(13)}$$

Where:
$\mathbf{ECE(U)_{ref}}$ is the reference unit value for avoided carbon dioxide emissions by the renewable plant, which:

- Must correspond to a unit value of carbon dioxide that would be emitted by a new production facility, which construction is avoided by the renewable plant;
- It's value is 2*10 – 5 EUR/g;
- It will be used, in each plant, during all periods in which the remuneration set by VRD is applied.

$\boldsymbol{CCR_{ref}}$**:** is the unit amount of emissions of carbon dioxide from the reference plant, which takes the value of 370 g / kilowatt-hour, and it will be used in each plant, during all periods in which the remuneration set by VRD is applied.

The parameter $\boldsymbol{LEV}$ for this project with a capacity less than 5 MW, takes the value of 0,035.

The factor **Z** is the technology used in production. The value for the mini-hydro is 4,5 and not 4,2 as indicated in the analysis.

$\boldsymbol{\frac{IPC_{m-1}}{IPC_{ref}}}$ was calculated taking into account the IPC of 2005 (the year preceding the project) and updated by inflation.

The following table represents the values determined for all parcels, together with the return of energy to the month and year:

Table 9 - Results of energy remuneration

KMHOm	*1*
IPCm-1/IPCref	1
LEV	0,035
PF(VRD)m	323,63

PV(VRD)m	3998,42
PA(VRD)m	821,90
Z(mini-hydro)	4,2
VRDm	**8.056 €**
VRDa	**96.672 €**

Source: Elaborated by the author

Another important note about the value of remuneration is the period of support. In this investment analysis it is considered that the energy generated will be paid during all the life of the project in this amount, however, this support has only the durability of 20 years, renewable for another five years, i.e. has a total 25 years of provision. Obviously, due to simplicity reasons in the calculation it is assumed that the revenues will be generated according to this value during the 50 years of the project, but this is not correct, since that in the middle of the project, energy will be sold according to market conditions, which provides a highest uncertainty and increases the risks for investment.

➢ Inflation rate

The inflation rate is assumed to be equal to 3%. However, this is only applied when calculating the remuneration of the energy produced, but it is not accounted for the remaining components of cash-flows, i.e. the cost of the project. As a result, this evaluation indicates that the projects revenues are growing over the years, but in return, the costs remain, which over-evaluates the NPV, benefiting positive results of project.

The inclusion of inflation in investment analysis is not consensual. Some authors argue that it only justifies to realize an evaluation at current prices if the inflation rate is very high and unstable, if not, a constant price analysis is best (Barros, 1991). This is common in studies of investment assessment, since it is considered that inflation affects in the same way all the revenues and costs. This situation happens due to the fact that many analysts, for simplicity, consider identical values of inflation for all components (Barros, 1991, Soares et al., 2008). This assumption is not realistic, since each component has different values of inflation, by types of products or sectors.

The inflation has an impact on cash-flows on investment projects at three levels (Soares et al. 2008):

- In nominal incomes, which increase;
- In nominal expenditures, they also increase;
- In the interest and charges relating to debt, which also increases.

For this reason, if this assessment considers inflation in revenues, it should also consider it in costs and in interest rates related to debt.

It is also important to note that the differentiated application of inflation is difficult to achieve and can lead to substantial errors. By analysing, for example, the replacement value of the transformer after 25 years, it is calculated based on weak assumptions, since it considers that it will cost five times more than its cost in the initial investment.

Regarding the choice between the evaluation at current or constant prices, the costs and profits reflect equally the impact of inflation, both investment analysis are equivalent, being the impact of inflation neutral. However, depreciations are determined by the underlying assets. Given that, these are accounted and remain at historical cost in corporate balance sheets, depreciation is a constant proportion of that cost, so it should not suffer the effect of inflation on an analysis at current prices. As depreciation is a cost that is not affected by price increases, but incomes reflect this growth, the impact of inflation will be an increase in net income before taxes and, by extension, a real increase in paid taxes. The real profitability of the company is reduced by the transfer of wealth from the company to the Government through higher taxes (Soares et. al., 2008).

In the case of determining the cash-flows at current prices, also the opportunity cost of capital, must be updated with inflation. Thus, the estimation of the discount rate, adjusting the effect of inflation, is the following relationship:

$$\boldsymbol{i_{Nominal} = i_{Real} + \pi + i_{Real} * \pi} \qquad \textbf{(14)}$$

Where:

$\boldsymbol{i_{Nominal}}$:Rate of capital cost at current prices

i_{Real}:Rate of capital costs at constant prices

π: Inflation rate

- **Depreciations**

In this evaluation is used a method of depreciation of constant quotas. However, there is some inconsistency about how it is applied. The following table represents the depreciation calculated:

Table 10- Depreciations by year

Year	*Assets*	*Depreciation*	*Accumulated depreciation*	*Net Value*
Start	810.000 €			
End of 2006	710.000 €	16.200 €	16.200 €	693.800 €
End of 2007	710.000 €	16.200 €	32.400 €	677.600 €
End of 2008	710.000 €	16.200 €	48.600 €	661.400 €
End of 2009	710.000 €	16.200 €	64.800 €	645.200 €
End of 2010	710.000 €	16.200 €	81.000 €	629.000 €
End of 2011	710.000 €	16.200 €	97.200 €	612.800 €
End of 2012	710.000 €	16.200 €	113.400 €	596.600 €
End of 2013	710.000 €	16.200 €	129.600 €	580.400 €
End of 2014	710.000 €	16.200 €	145.800 €	564.200 €
End of 2015	710.000 €	16.200 €	162.000 €	548.000 €
End of 2016	710.000 €	16.200 €	178.200 €	531.800 €
End of 2017	710.000 €	16.200 €	194.400 €	515.600 €
End of 2018	710.000 €	16.200 €	210.600 €	499.400 €
End of 2019	710.000 €	16.200 €	226.800 €	483.200 €
End of 2020	710.000 €	16.200 €	243.000 €	467.000 €

End of 2021	710.000 €	16.200 €	259.200 €	450.800 €
End of 2022	710.000 €	16.200 €	275.400 €	434.600 €
End of 2023	710.000 €	16.200 €	291.600 €	418.400 €
End of 2024	710.000 €	16.200 €	307.800 €	402.200 €
End of 2025	710.000 €	16.200 €	324.000 €	386.000 €
End of 2026	710.000 €	16.200 €	340.200 €	369.800 €
End of 2027	710.000 €	16.200 €	356.400 €	353.600 €
End of 2028	710.000 €	16.200 €	372.600 €	337.400 €
End of 2029	710.000 €	16.200 €	388.800 €	321.200 €
End of 2030	710.000 €	16.200 €	405.000 €	305.000 €
End of 2031	710.000 €	16.200 €	421.200 €	288.800 €
End of 2032	710.000 €	16.200 €	437.400 €	272.600 €
End of 2033	710.000 €	16.200 €	453.600 €	256.400 €
End of 2034	710.000 €	16.200 €	469.800 €	240.200 €
End of 2035	710.000 €	16.200 €	486.000 €	224.000 €
End of 2036	710.000 €	16.200 €	502.200 €	207.800 €
End of 2037	710.000 €	16.200 €	518.400 €	191.600 €
End of 2038	710.000 €	16.200 €	534.600 €	175.400 €
End of 2039	710.000 €	16.200 €	550.800 €	159.200 €
End of 2040	710.000 €	16.200 €	567.000 €	143.000 €
End of 2041	710.000 €	16.200 €	583.200 €	126.800 €
End of 2042	710.000 €	16.200 €	599.400 €	110.600 €
End of 2043	710.000 €	16.200 €	615.600 €	94.400 €
End of 2044	710.000 €	16.200 €	631.800 €	78.200 €
End of 2045	710.000 €	16.200 €	648.000 €	62.000 €
End of 2046	710.000 €	16.200 €	664.200 €	45.800 €
End of 2047	710.000 €	16.200 €	680.400 €	29.600 €
End of 2048	710.000 €	16.200 €	696.600 €	13.400 €
End of 2049	710.000 €	16.200 €	712.800 €	- 2.800 €
End of 2050	710.000 €	16.200 €	729.000 €	- 19.000 €
End of 2051	710.000 €	16.200 €	745.200 €	- 35.200 €
End of 2052	710.000 €	16.200 €	761.400 €	- 51.400 €
End of 2053	710.000 €	16.200 €	777.600 €	- 67.600 €

End of 2054	710.000 €	16.200 €	793.800 €	-	83.800 €
End of 2055	710.000 €	16.200 €	810.000 €	-	100.000 €

Source: Classes of Investments of Renewable Energy

In this context, the depreciations were calculated only for the tangible assets, taking into account the total value of the initial investment, with the exception of the components studies and projects, divided by the life time of the investment.

Meanwhile, some aspects deserve a special attention. First, it is assumed that all components of the investment are depreciated in the same way, which is not correct since, for example, the building has not the same lifetime tax of equipment. Thus, according to the rules of depreciation, it would be more appropriate to draw a map with the different amortization allocations for each component. Second, the component corresponding to land and expropriations is not depreciable, being only the value of the building included. Third, the studies and projects were not considered in the amortization map. However, although they represent intangible assets, these are amortized over three years. The following table presents an alternative to the amortization map:

Table 11 - Depreciations by components of investment

Components	***Depreciation (years)***	***Depreciation rates[3] (%)***
Equipment (Transformers, Generators, Turbines, Electromecanic Equipment)	16	6,25
Construction	30	3,33
Line of 15kV	20	5,00
Study and Project	3	33,33

Source: elaborated by the author

In addition, the component relative to equipment suffers a change in its value, due to the replacement of a transformer at the end of 25 years, which will increase both the value of fixed assets and depreciation in that year.

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[3] According to **Decreto Regulamentar n.º 2/90 de 12 de Janeiro**

- **Rate of Capital costs**

The rate of capital costs considered corresponds to 10%. This rate should be calculated according to the Weighted Average Cost Of Capital (WACC), determined as follows:

$$\boldsymbol{WACC = K_S * W_S + W_D * K_{D*} * (1 - T)} \quad \textbf{(15)}$$

Where:
K_S: Rate of return required by shareholders, promoters of the project;
W_S: Weight of equity;
W_D: Weight of debt;
K_D: Nominal interest rate;
T: Tax rate on profits

The WACC indicator shows the composition in terms of funding sources. The data for its calculation can be based on the historical balance sheets of the company or market values, being theoretically more correct the use of market values (Mithá, 2009). In the specific case of determining the Beta for the cost of equity, one of the major problems is that it is not possible to determine this value for companies not publically traded and, for this reason, the solutions given are for the use of the Beta of comparable companies; use average Beta of business related (*bottom-up*); use Beta of the listed companies with which there is strong correlation of activities (customers, suppliers, business sector).

- **NPV, IRR e Payback**

The results are a reflection of the assumptions taken into account in evaluating this investment. The value of NPV is low, considering the high investment, the payback is 35 years and the IRR is only 1% above the rate of capital cost considered.

Issues, such as not changing the depreciation values when the new equipment is incorporated in the mid-life of the project, not updating properly all investment components and the determination of little founded assumptions, makes these results less realistic.

One of the facts to comment in these results relates mainly to the calculation of the IRR. This project has non-conventional cash-flows, with signal change in more than one moment of its life time, which involves multiple IRR's and not just one, as shown. In this case, the solution in multiple IRR's is calculating the Modified IRR (IRRM) (Soares et. al., 2008).

First, it should upgrade to the invested capital for the time 0 (t0), to cost of capital. Then capitalize the successive operating cash-flows for the end of the life of the project (tn), to reinvestment rate that the company believes to have strong chances of getting, or ultimately, to a rate equal to the cost of capital. Finally, updates to the sum of the capitalized cash-flows for the time t0 at a rate (MIRR) that allows equals them to investment (Soares et al., 2008). Analytically, we have:

$$\frac{\sum_{t=1}^{n} OCF_t (1+R_2)^{n-t}}{(1+IRRM)^n} = CI_0 \tag{16}$$

Where:
OCF_t**:** Operating cash-flow at the end of year *t*;
R_2**:** Reinvestment rate of operating cash-flows;
$MIRR$: Modified Internal Rate of Return.
IC_0: Sum of investments in the project updated to rate of capital costs.

Using the MIRR is very useful in cases like these, it only allows associating a measure of profitability to a set of cash-flows (Soares et al., 2008).

Concluding this chapter, this analysis assumes three key assumptions that strongly affect the results of the evaluation:

1 - *The plant will produce to full capacity and all the energy produced will be sold during the life of project.*

The fact that it will produce at full capacity over 50 years is optimistic but also unrealistic, since these predictions are based on hydrological studies, considering the average annual water flow. The failure to consider the uncertainty in this case can be a mistake that could put the viability of the project at risk.

2 –Energy *remuneration is constant over 50 years of life.*

It does not seem reasonable to assume that the price of electricity will not change over 50 years. Even though, it is considered that the government will keep a constant remuneration for the energy produced, but this will only be valid for a maximum of 25, and not 50 years.

3 - *The rate of discount of 10% is assumed deliberately without consideration of funding sources.*

The discount rate definition is not normally consensual. However, in this case to assume a discount rate without any relation with the composition of funding sources is not correct. As mentioned in previous points, the discount rate has influence on the results of the NPV, thus, assuming rates that do not correctly evaluate the data for the project will produce incorrect results.

2.4. Methodology for Real Options application

To accomplish this evaluation through the ROA will be followed the following steps:

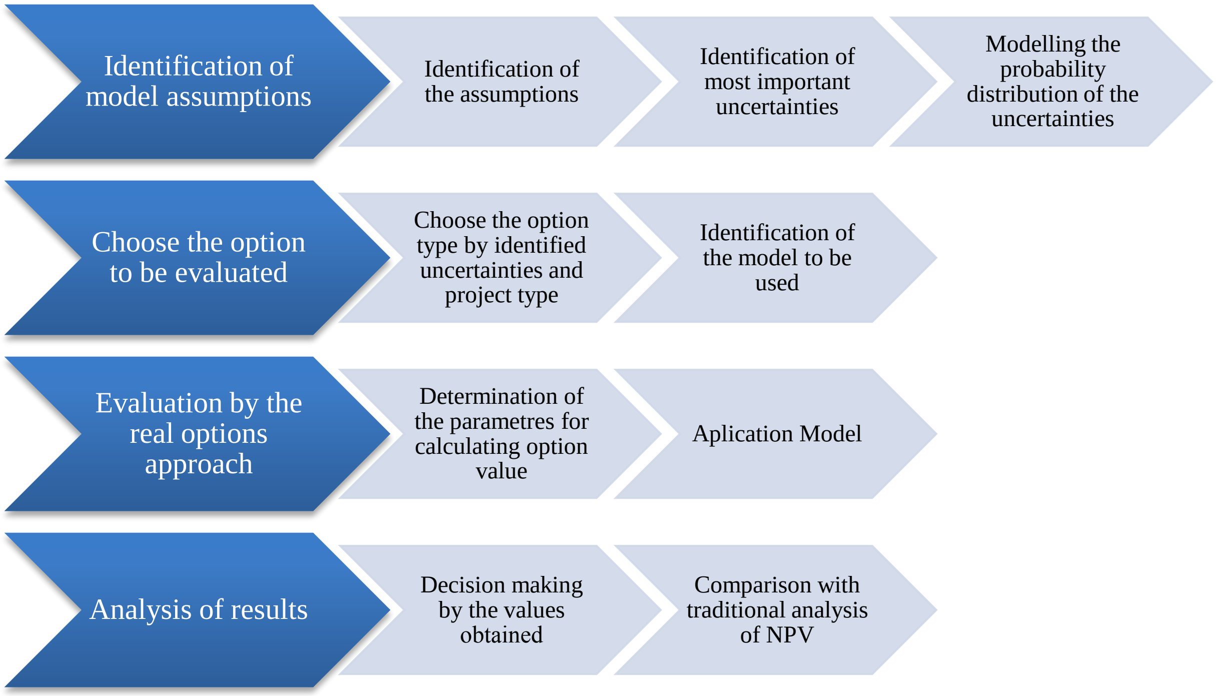


Figure 4 - Steps for Real Options Analysis
Source: Elaborated by the author

5.4.1. Assumptions

For development of investment analysis through ROA, it will be considered the following hypotheses:

Assumption 1 – All data provided by the traditional evaluation model and the results obtained are considered.

For the application of ROA, we assume the data provided by the project. This case study does not intend to alter any assumptions made by the traditional approach. Even being detected some weakness in this model, the objective of this study is to compare the traditional methods of project evaluation and the ROA. In addition, it would not be possible to make corrections accurately, because it is not possible obtain the necessary data to do these.

Assumption 2 – Considering the uncertainty about electricity prices

In spite of uncertainty dynamics that affect these projects, it will be only considered uncertainty of electricity prices, since in the case of mini-hydro investments, the operating costs are not affected by high levels of uncertainty. For example, in these cases, fuel costs have not a considerable influence on production costs. With regard to other uncertainties (technological change, environmental policies, among others), for simplicity case, they will not be included in the analysis.

For the modelling of this uncertainty, it will be considered the electricity price in long-term contracts in OMIP (the Iberian Power Derivatives Exchange) observed over four years.

In evaluation of long-term project of power generation, current spot price is not the most desirable for calculation of volatility project, because it may be strongly influenced by short-term factors (climate, availability of short-term production capacity, among others). In these situations, the uncertainty about time-average price over the lifetime of mini-hydro projects is more relevant.

Therefore, to calculate the volatility of investment return, it will be followed the premises of a GBM for modelling the probability distribution of long-term electricity prices. Pindyck (2001) discusses the evaluation of long-term commodity prices, and argues that for long-term investments related to energy (as the case of mini-hydro projects), the use of GBM will lead to small errors.

2.4.2. Modelling of uncertainties and Monte Carlo analysis

For project volatility assessment was applied a consolidated approach of uncertainty, defined by Copeland and Antikarov (2001), where all considered uncertainties on the assets value are combined into one uncertainty**:** the percentage of the project present value change over time, i.e., the investment return.

In the presented approach, the authors rely on the assumption that present value of cash-flows without flexibility is the best estimation of project market value, being for this reason considered as its market price. This value is used as an input in the binomial tree.

Copeland e Antikarov (2001) base their work on the theorem developed by Paul Samuelson (1965), which proves that the return rate of an asset follows a random trajectory, independently of the cash-flows generated in future, i.e., the current asset value already reflects all the information contained in the historical sequence of this asset. This implies that any deviation in the trajectory of future cash-flows will be given by random events, and consequently, the deviations on the rate of return will also be random.

Based on the ideas of Paul Samuelson, Copeland and Antikarov resorted to the method of Monte Carlo to combine several uncertainties in a single uncertainty, i.e., in volatility of return. The application of Monte Carlo simulation for calculating volatility of project return is represented as follows:

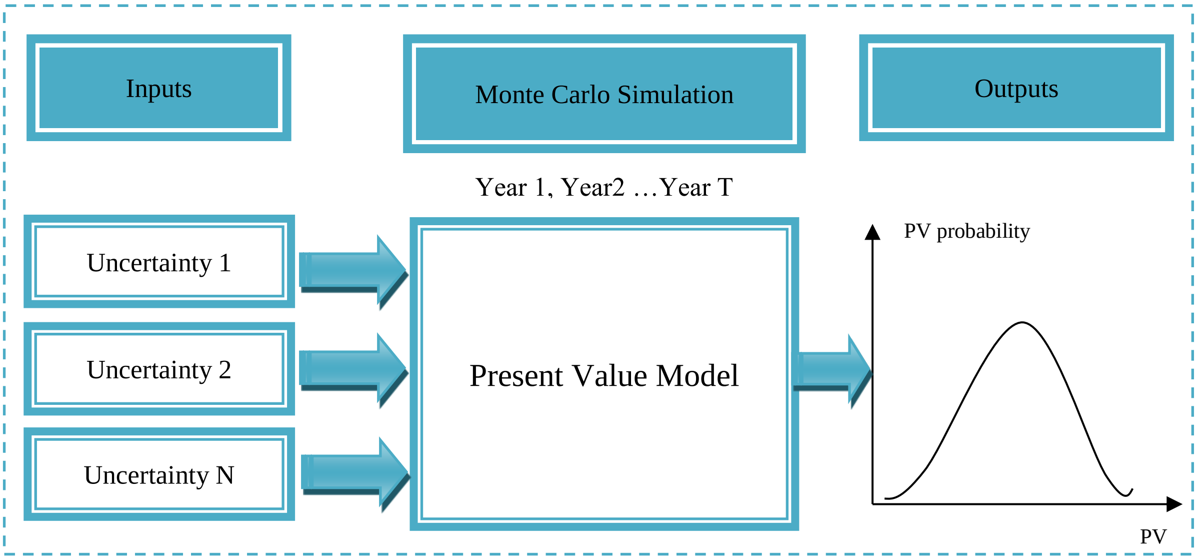


Figure 5 - Monte Carlo Simulation for calculating volatility of project return

Source: Elaborated by the author based on Copeland e Antikarov, 2001

These authors consider that the volatility of the project corresponds to the volatility of its returns. Thus, the values obtained in the simulation can be converted in a return rate by the following equation:

$$\boldsymbol{rt = \ln\left(\frac{PV_t}{PV_0}\right)} \quad \textbf{(172)}$$

Where:

PV_t: Present value at time t;

PV_0: Present value at time zero;

rt: Rate of return.

The value of future cash-flows are estimated for two dates, and given that the rate of return is constant over time, it is considered that *t* assumes the value one (*t=1*). Thus, the percentage change the project value of one period to the next can be calculated using a logarithmic scale as follows:

$$\boldsymbol{z = \ln\left(\frac{PV_1 + FCF_1}{PV_0}\right)} \qquad \textbf{(18)}$$

Where:

PV_1: Present value of project at time 1;
FCF_1: Free cash-flow at time 1;
PV_0: Present value of project at time 0.

The present value of the project at date 0 and date 1 can be calculated using the equations **(19)** and **(20)**, respectively:

$$\boldsymbol{PV_0 = \sum_{t=1}^{T} \frac{FC_t}{(1+WACC)^t}} \qquad \textbf{(19)}$$

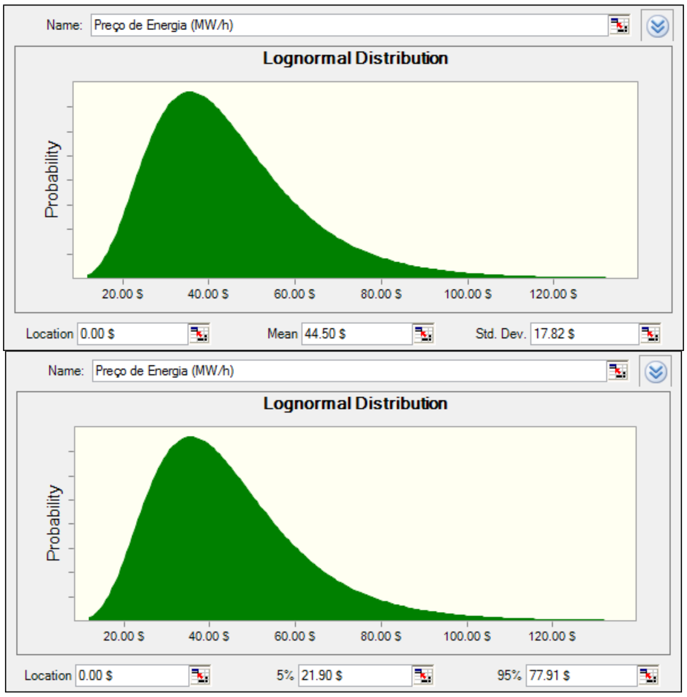
$$\boldsymbol{PV_1 = \sum_{t=2}^{T} \frac{FC_t}{(1+WACC)^{t-1}}} \qquad \textbf{(20)}$$

The probability distribution of the "z" values is obtained through the Monte Carlo simulation, thought the usage of the Crystal Ball software. During the simulation the denominator of the equation **(18)** $(\boldsymbol{PV_0})$ remains fixed, only varying $PV_1 + FCF_1$according to the uncertainties defined as *Assumption*. The project volatility is defined as the standard deviation of "z" in the following equation:

$$\boldsymbol{\sigma = desv.pad\,(z)} \qquad \textbf{(21)}$$

In this case, the values are: $VP_0 = [WACC; FC_1{:}FC_9] = 881.371€$ and $VP_1 = [WACC;\ FC_2{:}FC_9 = 851.422€]$. As a result, the value of *z* will be 9,53%.

As mentioned in the model assumptions, it was considered uncertainty on electricity prices in the Iberian market (OMIP) of long-term contracts. The price, because it cannot be negative, follows a lognormal distribution, being one of the premises of GBM. For this distribution, were defined the following values in the confidence interval of 5% to 95%: 21.90€ and 77.91€, corresponding to the lowest and highest price obtained in market over four years. The mean and standard deviation were calculated automatically by the program, giving values of 44.50 and 17.2 respectively. The following figures represent this procedure:


Name: Preço de Energia (MW/h)

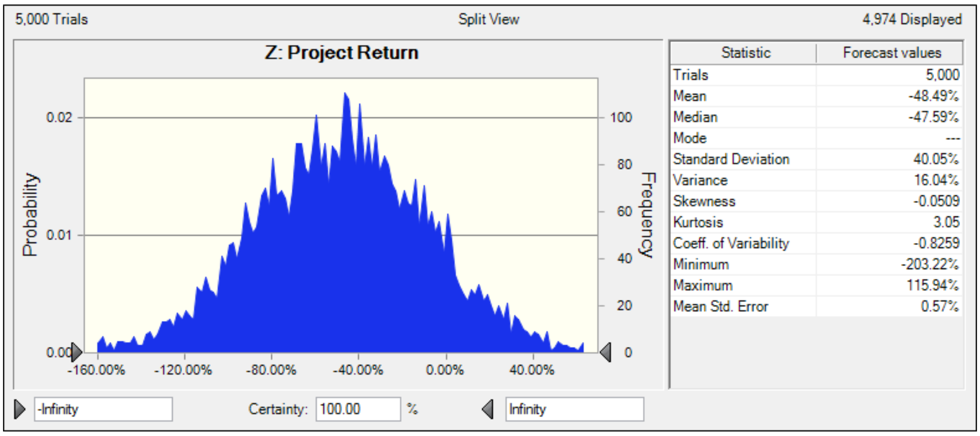
Location 0.00 $ Mean 44.50 $ Std. Dev. 17.82 $

Name: Preço de Energia (MW/h)

Location 0.00 $ 5% 21.90 $ 95% 77.91 $

Figure 6 - Distribution of electricity prices
Source: Elaborated by the author using Crystal Ball software

Then, it was defined the value of *z* as *Forecast*, and proceeded to a Monte Carlo simulation with 5000 iterations, obtaining a standard deviation of project returns of approximately 40%, which corresponds to the volatility of the project (see **Figure 7**).


5,000 Trials Split View 4,974 Displayed

Statistic	Forecast values
Trials	5,000
Mean	-48.49%
Median	-47.59%
Mode	---
Standard Deviation	40.05%
Variance	16.04%
Skewness	-0.0509
Kurtosis	3.05
Coeff. of Variability	-0.8259
Minimum	-203.22%
Maximum	115.94%
Mean Std. Error	0.57%

-Infinity Certainty: 100.00 % Infinity

Figure 7 - Forecast of project returns
Source: Source: Elaborated by the author using Crystal Ball software

2.4.3. Modelling Real Options

It is important to emphasize that the investment on a mini-hydro, with exception of the study phase, is not implemented in phases. In other words, once the project starts, it is unlikely to hold an option to interrupt the plant (Fenolio and Minardi, 2008).

Therefore, in this case study, it will be studied the option of deferring the project within five years. This hypothesis of postponement is justified by the high uncertainty on regulatory change that may arise. In other words, given the current economic crisis, the government believes that the support given to electricity generation from renewable sources is no longer a priority and that the legislation could be changed in the coming years, conditioning the feasibility of these projects. Thus, the remuneration of the new plants would no longer have a constant remuneration, being subject to the uncertainty of electricity prices on the open market.

Given this uncertainty, it will be evaluated through the ROA, the option of delaying construction for a maximum period of five years for to obtain better information about new legislation and price evolution, and the option of investing now.

Thus, as presented in the previous point, a deferral option corresponds to an American call option, in which the decision to invest now will be taken if the NPV of the project exceeds the value of the option to defer.

In this case, it is applied the binomial tree method developed by Cox, Ross and Rubinstein (1979), in which the parameters found for the construction of the tree are represented in the following table:

Table 12- Parameters for binomial tree construction

Stock Price (S)(€)	881.371
Exercise price (k)(€)	830.000
Time to option expiration (days) (T)	1.825
Volatility(σ)	0,40
Risk-free rate (rf)	0,07
Number of steps (n)	5
$\Delta T=(T/365)/n$	1
$\mu=\exp(\sigma\sqrt{\Delta T})$	1,49
$d=1/\mu$	0,67

exp(rf*ΔT)	1,07
p=(exp(rf*ΔT)-d)/(u-d)	0,49

Source: Elaborated by the author

As previously stated, the stock price represents the cash-flows of an investment and exercise price is the investment required to implement the project. The time to maturity of the option to defer is 5 years and the volatility of investment returns found by the method of Monte Carlo simulation is approximately 40%.

The risk-free rate of return considered represents the rate of return on Treasury bonds to 10 years. The coefficients of ascent and descent of the underlying asset's values μ and d (Equations (14) and (15))assume values of 1.49 and 0.67, respectively. Finally, the value of probability of the underlying asset price increases is 49%, while the probability of decreasing assumes a value of 51%.

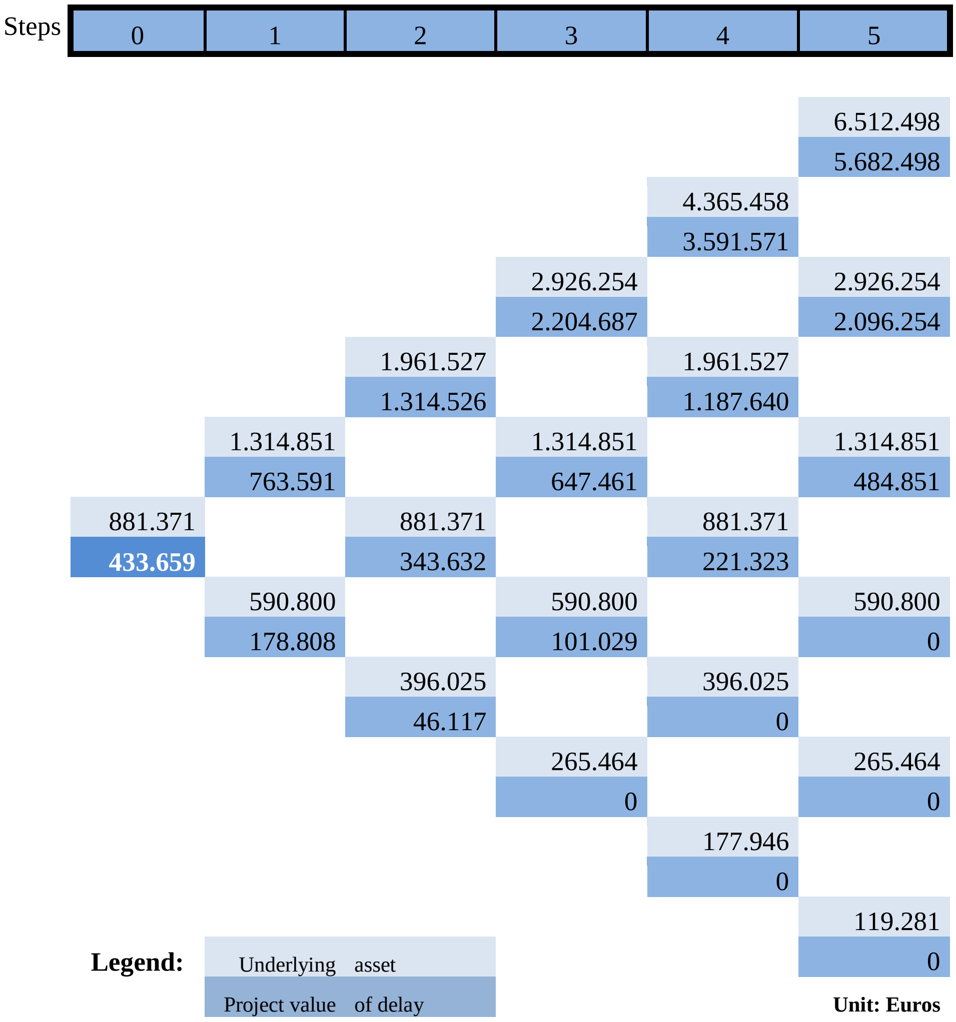
Determined these variables the tree is constructed with the possible evolutions of the underlying asset price from left to right, being placed in the node on extreme left the current price of the underlying asset. At each time interval, the price can increase or decrease depending on the coefficients μ and d, respectively. The last column of the binomial tree represents the possible values of the underlying asset at the maturity of the option.

After, it is elaborated an evaluation tree of the option from right to left. Given that the abandon option is a call option, from the values of the last column of the underlying asset is subtracted to each one of these values the exercise price (investment on the project), and this result takes the max value between S-K and 0. To determine the remaining values of the evaluation of the call option, it is applied the neutral probability to each pair of vertically adjacent values.

2.4.4. Results

The main issue of this evaluation is to determine if the investment of this mini-hydro should be performed immediately, or if it should be deferred up to five years for to obtain better information about changing the remuneration of these plants. Thus, if the value obtained for the project with the option to delay is greater than the value derived from the investment without considering flexibility, the decision more advisable will be to exercise the option.

The results obtained in the binomial tree in relation to the future underlying asset values and values of the project with the option of postponing are represented in the upper and lower values, respectively, in each node in the following figure:



Steps	0	1	2	3	4	5
						6.512.498
						5.682.498
					4.365.458	
					3.591.571	
				2.926.254		2.926.254
				2.204.687		2.096.254
			1.961.527		1.961.527	
			1.314.526		1.187.640	
		1.314.851		1.314.851		1.314.851
		763.591		647.461		484.851
	881.371		881.371		881.371	
	433.659		343.632		221.323	
		590.800		590.800		590.800
		178.808		101.029		0
			396.025		396.025	
			46.117		0	
				265.464		265.464
				0		0
					177.946	
					0	
						119.281
						0

Legend: Underlying asset / Project value of delay

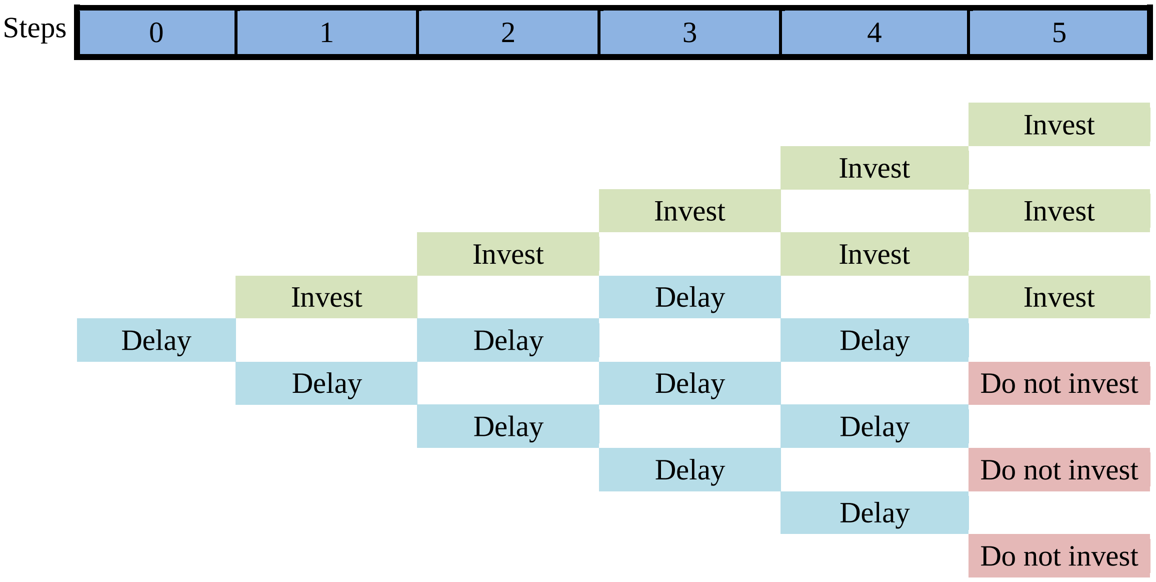
Unit: Euros

Figure 8 – Evolution of underlying asset and project value of delay
Source: Elaborated by the author

The calculated value of the project with the option to delay is €433.659, much higher than the static NPV, which was €51.371. The option value of delay is obtained from the equation (8), i.e. the difference between static NPV and expanded NPV, resulting in a value of €382.289. Therefore, it is appropriate to postpone the project, because the option value of delay is much higher than the NPV of investing immediately.

With the investment and current revenues constrained by the electricity price (S = € 881,371) and a volatility of 40% (σ = 0.4), the option to postpone the investment has value and should be exercised. For this reason, there is value in waiting for more favourable conditions for investment.

This conclusion is based on the premise that, since the investment decision involves a loss of opportunity to defer this decision, the investment should be undertaken only when its NPV exceeds the value of the deferral option (Soares et al., 2008). This happens because investing now implies that there is a missed opportunity to wait for more information about the evolution of electricity remuneration, which corresponds to the value of the option to defer. Therefore, it is not enough that the value generated by the project covers the investment, but it also should be sufficiently high to cover the option of delaying the project. Under this assumption and since that this assessment is realized in continuous time and the option to invest now or delay can be taken at any time during the interval of five years, it is determined the following decision tree from the values found:



Steps	0	1	2	3	4	5
						Invest
					Invest	
				Invest		Invest
			Invest		Invest	
		Invest		Delay		Invest
	Delay		Delay		Delay	
		Delay		Delay		Do not invest
			Delay		Delay	
				Delay		Do not invest
					Delay	
						Do not invest

Figure 9 - Decision Tree
Source: Elaborated by the author

As empathized by Dixit and Pindyck (1994), the option to defer an investment for a time t +1 can be seen as the opportunity cost of investment. Investing in time t, means that we are throwing away the option to defer and the company must pay the opportunity cost and also the initial investment. Thus, for that project to be accepted at time *t*, it is not enough that the present value of the cash-flows are positive, as established in the traditional NPV rule, but it also has to be sufficiently positive to exceed the initial investment in an amount equal to the opportunity cost.

Therefore, as shown in the decision tree (**Figure 9** - Decision Tree), in each time interval can be assessed which option maximizes the value of the project, by looking at the upward or downward trend of the underlying asset. This decision is based on maximum value between the static NPV and the option value of delaying the project for each node. As we can seen, for higher values of the underlying asset, the best option is to invest now, and for lower values of the underlying asset, the option value of postponing the project for the next period is more valuable.

In other words, since the underlying asset value depends mainly on the electricity price, the investor will choose to invest if the evolution of energy remuneration is sufficient to overcome the investment costs and the opportunity cost of not postponing the project.

In this particular case, the project despite having a positive outcome, presents a low static NPV, given the high investment and lifetime of the project. In other words, is necessary to invest € 830.000 to implement the project now obtaining € 51.371 after 35 years. Thus, even intuitively, any investor would prefer to wait for more information and minor uncertainty. This decision tree shows that even when the static NPV is positive the project is delayed, because the value of the deferral option is superior.

In the last year (step 5), the investor will no longer be able to postpone the project, so he must to decide if the conditions are favourable for investing in that moment, or if the project will not be implemented. At the end of the expiration of the deferral option, he will only invest if electricity prices are sufficiently high, otherwise, the investor will choose not to invest.

To postpone the project increases its value. This happens because during the waiting period uncertainty about the economy has been resolved, and this information allows a better decision. Obviously, delay also involves losses, in terms of cash-flows that are lost, and in terms of competition. For this reason, this type of analysis should be performed with special care, because in an uncertainty context, not to include losses of project postponement, could mean never investing in the project due to the gains of more information and consequent reduction of uncertainty.

For the traditional NPV analysis, not considering the value of flexibility, underestimates the project value. The assessment by the ROA gives the investor flexibility to re-evaluate the project in future stages, and from that information, redefine his strategy.

With the incorporation of Real Options in the analysis, it is possible to show that the NPV of the project increases in the considered period, confirming the premise that a project that can be delayed has more value than one without flexibility to delay, given that the investor has the option to defer the start of the project, taking into account the risks and the possibility of change.

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